STAT 101 Spring 2012
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Chapter 16 – Random Variables

Terminology
probability distribution
random variable
expected value (aka mean)
standard deviation & variance

Find expected value of a random variable based on a given distribution
$E(X) = x_1*P(x_1) + x_2*P(x_2) +\ldots+ x_n*P(x_n)$
Find expected value of a game with a cash prize
a "fair price" for a random game
Compute probabilities for a distribution
Create a probability distribution based on a problem
Compute standard deviation for a probability distribution
Formula for variance & standard deviation
$Var(X) = E[(X – E(X))^2) = E(X^2) – E(X)^2$
$\sigma = \sqrt{Var(X)}$

Effect of operations on random variables to Mean and Standard Deviation (for independent random variables)
$E(3X) = 3*E(X)$
$E(X+4) = E(X) + 4$
$E(X+Y) = E(X) + E(Y)$

$Var\ (X+5) = Var(X)$
$SD(X+ 5) = SD(X)$
$SD(3X) = \sqrt{Var(3X)} = \sqrt{9*Var(X)} = 3*SD(X)$
$Var(X+Y) = Var(X) + Var(Y)$
$SD(X+Y) \neq SD(X) + SD(Y)$
$SD(X+Y) = \sqrt{Var(X+Y)} = \sqrt{(Var(X) + Var(Y))} = \sqrt{(SD(X)^2 + SD(Y)^2)}$
SD(X – Y) = SD(X + Y)!! You still add – the SD only gets bigger.

Calculating Standard Deviation from a Probability Distribution:
Method 1: $\sqrt{(E((x-E(X))^2))}$

x	P(X=x)	x-E(X)	$(x-E(X))^2$
19	0.4	0.6	0.36
5	0.3	-13.4	179.56
27	0.2	8.6	73.96
39	0.1	20.6	424.36
	E(X) = 18.4		$E((x-E(X))^2) = 111.24$ $\sigma = \sqrt{111.24} =$ **10.5470**

Or use $\sqrt{(E(X^2) – E(X)^2)}$

x	P(X=x)	x^2	
19	0.4	361	
5	0.3	25	
27	0.2	729	
39	0.1	1521	
	E(X) = 18.4	$E(X^2) = 449.8$	$E(X^2) - E(X)^2 = 449.8-338.56 = 110.68$ $\sigma = \sqrt{110.68}$ **10.5205**

Why are they different? This is due to rounding error in the first method.